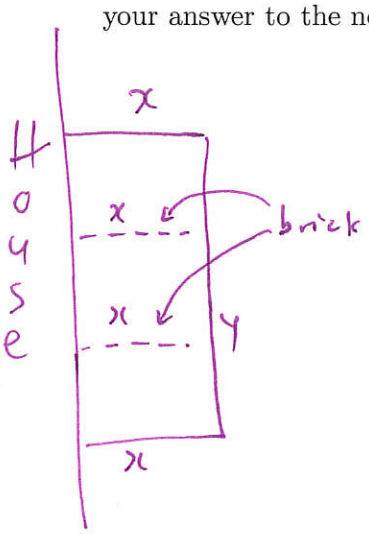
Quiz 5, Calculus I

Dr. Adam Graham-Squire, Fall 2017

Name: Key

1. (4 points) Henrietta decides to build a rectangular garden next to her house. One side of her garden is up against the house, and she will put up fencing around the other three sides (two widths and one length). She will also separate the garden into three rectangular subsections by laying down two lines of bricks width-wise across the garden. The exterior fencing costs \$5 per linear foot, and the bricks cost \$2 per linear foot. If Henrietta has \$1000 to spend on the garden, *use calculus* to find what the maximum area she could enclose would be. Round your answer to the nearest square foot.



$$\text{Cost} = 5(2x+y) + 2(2x)$$

$$1000 = 10x + 5y + 4x$$

$$1000 = 14x + 5y$$

$$\Rightarrow \frac{1000 - 14x}{5} = \frac{5y}{5}$$

$$200 - \frac{14}{5}x = y$$

$$\text{Area} = xy$$

$$A(x) = x\left(200 - \frac{14}{5}x\right)$$

$$A(x) = 200x - \frac{14}{5}x^2$$

$$A'(x) = 200 - \frac{28}{5}x$$

$$0 = 200 - \frac{28}{5}x$$

$$\frac{5}{28}(200) = x$$

$$x = 35.7143$$

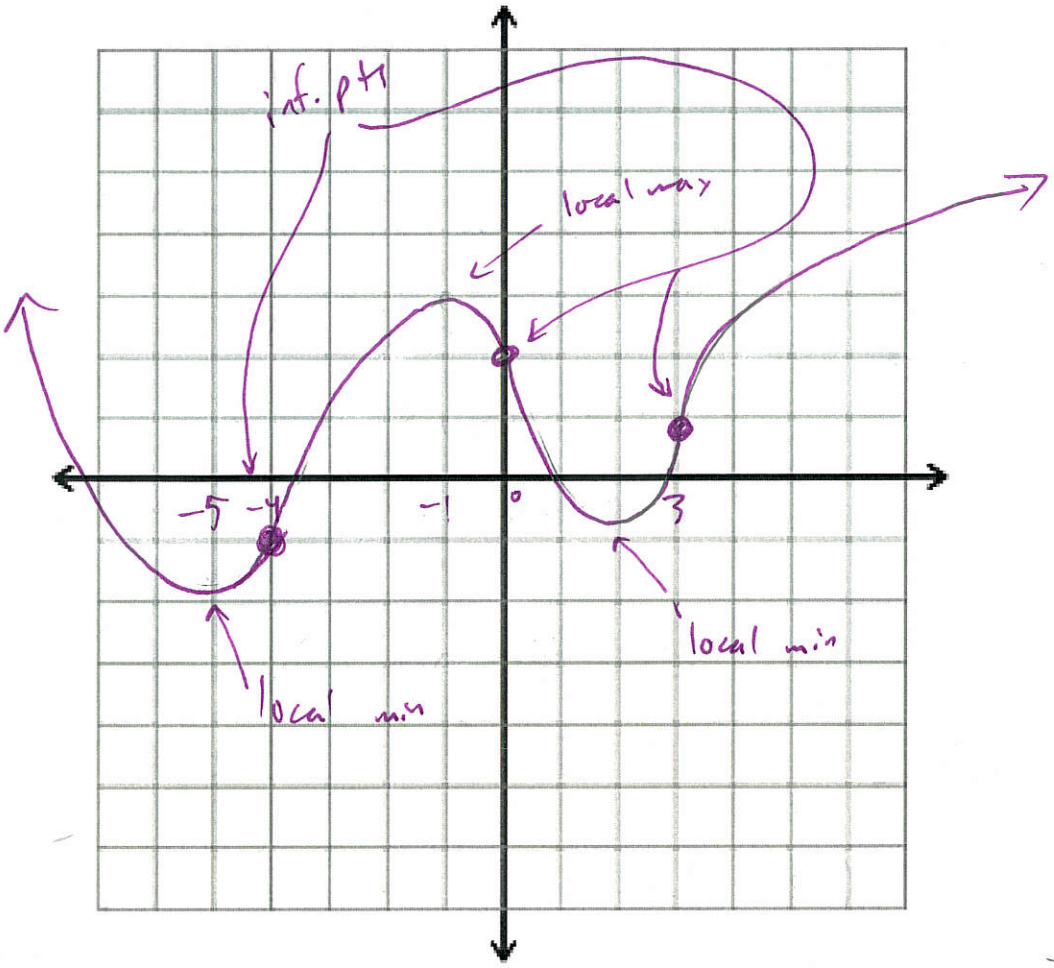
$$\Rightarrow A(35.7143) = 3571.43$$

$$= \boxed{3571}$$

2. (3 points) Let f be a function such that $f''(x) = \dfrac{x^2 + x - 12}{x^{1/3}}$. We also know the following:

- f is ~~increasing~~ dec on the intervals $(-\infty, -5), (-1, 0)$ and ~~$(0, \infty)$~~ (2,3) (3,∞).
- f is ~~decreasing~~ incr. on the interval $(-5, -1)$,
- f ~~goes through~~ has local max at the point $(-1, 3)$.

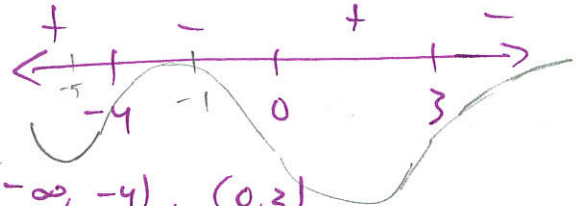
Calculate the intervals of concavity and sketch a graph of f. Label all the maximums, minimums, and inflection points.



$f''(x) = 0$ when $0 = x^2 + x - 12 \Rightarrow 0 = (x+4)(x-3)$

$x = -4$ $x = 3$

$f''(x)$ dne when $x = 0$



$f''(-5) = -$

$f''(-1) = +$

$f''(1) = -$

$f''(4) = +$

$\Rightarrow$ conc. up on $(-\infty, -4), (0, 3)$

" down " $(-4, 0), (3, \infty)$

3. (3 points) Calculate the limit:

$$\lim_{x \to \frac{\pi}{2}} [\tan(3x)\cos(15x)]$$

$\to \tan\left(\frac{3\pi}{2}\right) \cdot \cos\left(\frac{15\pi}{2}\right)$

$= \pm\infty \cdot 0$ ind!

$$= \lim_{x \to \frac{\pi}{2}^+} \frac{\sin(3x) \cdot \cos(15x)}{\cos(3x)}$$

$$\overset{L}{=} \lim_{x \to \frac{\pi}{2}^+} \frac{3\cos(3x)\cdot\cos(15x) + \sin(3x)\cdot 15(-\sin(15x))}{-3\sin(3x)}$$

$$= \frac{3\left(\cos\frac{3\pi}{2}\right)\cdot\cos\left(\frac{15\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right)\cdot 15\cdot\sin\left(\frac{15\pi}{2}\right)}{-3\sin\left(\frac{3\pi}{2}\right)}$$

$$= \frac{0\cdot 0 - (-1)\cdot 15\cdot(-1)}{-3(-1)}$$

$$= -\frac{15}{3} = \boxed{-5}$$